Events involving NOT, OR, AND

$P(not\ E) = 1 - P(E)$.

To find *P(A* OR *B)*, we use $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
Divide by *n(S)* and get
P (A OR *B) = P(A) + P(B) –P(A* AND *B).*

Conditional probability.

Let *A* and *B* be sets.
***P(B|A)* is the probability of *B* given *A*.**

E.g., *S = {1,2,...,10}*.
A = set of odd elements from *S*
B = set of multiples of 3 from S
P(B|A) is the probability of choosing a multiple of 3 from *S* where you chose an odd element.

$P(B \mid A) = \dfrac{n(A \cap B)}{n(A)}$, as this is the number of both *A* and *B* divided by the number of *A.*

Divide numerator and denominator by $n(S)$. We get

$$P(B \mid A) = \frac{P(A \text{ AND } B)}{P(A)}.$$

Likewise, since *A* and *B* could be anything:

$$P(A \mid B) = \frac{P(A \text{ AND } B)}{P(B)}.$$

If *A* and *B* are independent, then
P(B | A) = P(B), as the probability does not depend on *A*. We get
$P(A \text{ AND } B) = P(B)P(A)$ if they are independent.
This is the multiplication rule.

Summary for independent probabilities:
AND multiply probabilities. OR add probabilities.

When a doctor suggests surgery, ask two questions:

1. Out of 100 people, how many do well? Note: Do not ask for the probability, which is a fraction <= 1, but say out of 100 people. If you did this procedure on 100 people, how many would do well?

2. For those who did not do well, what was their condition? E.g., if 95 out of 100 would do well, what is the situation of the other 5?

Examples of conditional probability

A doctor gives a patient a test for a particular cancer. Before the results of the test, the only evidence the doctor has to go on is that 1 woman in 1000 has this cancer. Prior(cancer) = 0.001.

Experience has shown that, in 99% of the cases in which cancer is present, the test is positive. In 95% of the cases in which cancer is not present, the test is negative. This means that 5% of cancer-free people test positive.

If the test turns out to be positive, what probability should the doctor assign to the event that cancer is present?

Let us look at 100,000 people, for thinking about people is easier than thinking about percentages.

100 are sick. Out of the 100 sick people, 99 test positive.

Out of the 99,900 healthy people, 5%, or 4995, tested positive.

The sample size, the number of positive, is 99 + 4995, or 5094.

The probability of someone testing positive being sick is 99/5094, or 0.0194. This means out of 100 people tested positive, almost 2 will be sick. In other words, the conditional probability that you have the disease if you test positive is 1.9%.

We see now that the probability of cancer given a positive test has only increased from .001 to .019. While this is 19-fold increase, the probability that the patient has the cancer is still small. Stated in another way, among the positive results, 98.1% are false positives, and 1.9 % are cancers.

When a group of second-year medical students was asked this question, over half of the students incorrectly guessed the probability to be greater than 0.5.

Another example. In Life Tables, one finds that in a population of 100,000 females, 89.835% can expect to live to age 60, while 57.062% can expect to live to age 80. Given that a woman is 60, what is the probability that she lives to age 80?

This is an example of a conditional probability. In this case, the original sample space can be thought of as a set of 100,000 females. Let E be the subset of the sample space consisting of all women who live at least 60 years, and F the subset of all women who live at least 80 years. We note that $F \subseteq E$.
$n(E) = 89{,}835$ and $n(F) = 57{,}062$.

$$P(F \mid E) = \frac{57{,}062}{89{,}835} = 0.6352.$$

Thus, a woman who is 60 has a 63.52% chance of living to age 80.

Binomial probability
We use the multiplication rule for binomial probability.

Let p be the probability of success, and q the probability of failure, where p and q are independent. Since they are independent, we can use the multiplication rule.

Probability of x successes out of n trials, if the first x trials are successes, is $p^x q^{n-x}$.
This uses the multiplication rule, as the probabilities are independent.
Note that x is a whole number.

If we just care for x successes, but do not care if the x successes are first or wherever, then we multiply by the number of ways of choosing x items out of n where order does not matter. This is ${}_nC_x$. This gives the binomial probability formula p 759:

$$P(x) = {}_nC_x p^x q^{n-x}.$$

E.g., find the probability of getting one head in two tosses of a fair coin.
$n = 2, x = 1.$

$$P(1) = {}_2C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$

The sample space is $\{HH, HT, TH, TT\}$.

We see the probability is $\frac{2}{4} = \frac{1}{2}$.

Find the probability of a family with 5 children having 4 girls.
Using the binomial probability formula, we have

$$P = {}_5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5}{32}.$$

Let us write it down. We have 32 families, 5 of which have 4 girls.
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gggbg 2
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ggbgg 3
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bgggg 5
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Dr. S. Aranoff 12/3/2010