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The discussion is confusing with the subscripts. Here it is written clearly. When you read, if it is confusing, rewrite it yourself as I did here.

Definition:
$f(x_i), i = 1, \ldots, N$ is homogenous of degree k if $f(\lambda x_i) = \lambda^k f(x_i)$.

Volume is homogenous of degree 1:
If T, P are constant, then $V(\lambda n_i) = \lambda V(n_i)$, as $k = 1$, where n_i are amounts of substances.

T is homogenous of degree 0.

Euler's Theorem:

Theorem:

If $k = 1$, then $\boxed{x_i \frac{d(f(x_i))}{dx_i} = f(x_i)}$

Proof.

Let $f(\lambda x_i) = \lambda^k f(x_i)$.

$\frac{d(\lambda^k f(x_i))}{d\lambda} = k\lambda^{k-1} f(x_i)$, as $f(x_i)$ is not a function of λ.

$= \frac{d(f(\lambda x_i))}{d\lambda}$, as $\lambda^k f(x_i) = f(\lambda x_i)$

$= \frac{\partial f}{\partial x'_i} \frac{\partial x'_i}{\partial \lambda}$, where $\lambda x_i = x'_i$.

$= \frac{\partial f}{\partial x'_i} x_i$.

$\therefore x_i \frac{\partial f}{\partial x'_i} = k\lambda^{k-1} f$.

Let $\lambda = 1$. Then $x'_i = x_i$.

$x_i \frac{\partial f}{\partial x_i} = kf$.

Let $k = 1$ (e.g., for volume).

This proves the theorem.

For our case, let $f = \bar{V}$. Let $x_i = n_i$. We have

$$n_i \left(\frac{\partial \bar{V}}{\partial n_i} \right)_{T,P,n_j} = \bar{V}$$

$$= n_i \bar{V}_i, \text{ where}$$

$$\bar{V}_i = \left(\frac{\partial \bar{V}}{\partial n_i} \right)_{T,P,n_j}$$

Tricky steps: We set $\lambda = 1$, and then, later, $k = 1$.

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