Proof of the Euclidean algorithm :

We wish to find the GCF of two natural numbers a and b.
A factor common to the two numbers will also be a factor to the difference.
Look at the GCF of $(a-b), b$. That is, subtract the numbers to get another pair.
Repeat, subtracting the smaller from the larger
until we get a number $<=$ the smaller of the original numbers.
This number is the largest common factor.

We can write a computer program that you can program into your programmable calculator.

```
function GCF(a,b)
// Given two natural numbers it finds the Greatest Common Factor
while a > b do
   if a := a − b then
      a := a − b  // This means replace a with a − b. Like the STO button on a calculator.
   else if a = b then
      break
   else begin
      d := b;  // Store b temporarily
      b := b − a;  // Make b smaller.
      a := d;  // a is the original b before we changed it.
   end;

GCF := a
```

E.g, find GCF of 81 and 216.

$216-81=135$	We subtract the smaller from the larger.
$135-81=54$	We take the smaller two numbers, and subtract the smaller from the larger.
$81-54=27$	
$54-27=27$	

GCF is 27.

We do not have to divide 216 by 81, but we can subtract, which is easier.

Another example: 455, 130

$455 - 130 = 325$

$325 - 130 = 195$

$195 - 130 = 65$

$130 - 65 = 65$

GCF is 65.

To check:

$455 \div 65 = 7$

$130 \div 65 = 2$

Another example: 168, 90

$168 - 90 = 78$

$90 - 78 = 12$

$78 - 12 = 66$

$66 - 12 = 54$

$54 - 12 = 42$

$42 - 12 = 30$

$30 - 12 = 18$

$18 - 12 = 6$

$12 - 6 = 6$

GCF is 6.

In this case, the division approach is faster.

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