Complex numbers

Let us assume that the equation $x^2 = -1$ has a solution.
There is no real number that satisfies this equation.
Let us invent a number, and call it i, where $i^2 = -1$.
Since these assumptions are consistent with the rest of mathematics, we can accept them.

Let a be a real number. $a \in R$, where R is the set of real numbers.
We can define numbers ia. These are called **imaginary numbers**.
Let $b \in R$. We can then write numbers like $a + ib$.
Numbers like $a + ib$ are called **complex numbers**.

There is a theorem which states that every equation in one variable, no matter how complicated, can be solved using complex numbers.
We therefore do not need a third type of number.

Powers of imaginary numbers :

$i^3 = -1 \times i = -i$

$i^4 = (i^2)^2 = (-1)^2 = 1$

Since we need two quantities to define a complex number, we can identify complex numbers as points on a plane, with the x-axis the real numbers and the y-axis imaginary numbers.

There are more things that we can say. I'll just write them, but not hold you responsible.
$(a + ib)(a - ib) = a^2 + b^2$. Just expand it, and you'll see.
The distance from a point in the complex plane to the origin is called the **absolute value**.
$| a + ib | = \sqrt{a^2 + b^2}$.

Here is something that we will not prove:

$\cos\theta + i\sin\theta = e^{i\theta}$

This equation is one of the basic equations needed for engineering, such as power generation and transmission.
If $\theta = \pi$, then $e^{i\pi} = -1$.

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