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Basic assumptions and black holes

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Abstract: Science develops by initially proposing hypotheses to explain phenomena, then creating a formal consistent mathematical framework, verified by experiment and observation, called a scientific theory. This theory must exclude solutions that violate basic physical principles or yield inconsistent mathematics. New observations may challenge the theory, but do not necessarily invalidate it as a basic mathematical inconsistency would. Science continues by developing hypotheses to explain what the current theory cannot. The paper discusses objects falling into black holes using general relativity as examples of both valid theory and developing hypotheses. The existence (according to most physicists) of a black hole is part of a valid theory. The crossing of the event horizon of the black hole is an extension of the theory, but is not part of the theory, for the mathematics is not consistent, and the results violate basic principles. Another example of an invalid extension of a theory is given from special relativity. Since this theory is easier to understand, it helps clarify the point. One needs to be clear what is part of the theory, and what is not part of the theory. The statement "an observer crossing the event horizon and losing all contact with the rest of the universe is part of physics" is a counterexample to the proper understanding of theories. Statements incorrectly based on theories may be obstacles to the advance of science. One needs to understand the philosophical underpinnings of science, and how one can tease out the hidden assumptions of a theory. © *2009 Physics Essays Publication*. [DOI: 10.4006/1.3241135]

Résumé: La science se développe en proposant tout d'abord des hypothèses pour expliquer les phénomènes, puis ensuite en créant un cadre mathématique adéquat, validé par le biais de l'expérience et de l'observation. Ce cadre rigoureux devient alors synonyme d'une théorie scientifique. Cette théorie se doit cependant d'exclure les solutions qui mettent en porte-à-faux les principes physiques élémentaires ou encore celles qui donnent la primauté à une mathématique inconsistante. De nouvelles observations peuvent venir remettre en question la théorie, mais elles ne la rendent pas nécessairement nulle comme le ferait une mathématique incohérente. En réalité, la science se construit en développant des hypothèses à même d'expliquer ce qui ne peut l'être par la théorie actuelle. Le présent article traite des objets qui entrent dans la catégorie des trous noirs en utilisant le concept de la relativité générale comme exemple tout à la fois d'une théorie valide et d'hypothèses en cours de construction. L'existence (selon la plupart des physiciens) d'un trou noir fait partie intégrante d'une théorie valable. La traversée de l'horizon de l'évènement du trou noir est une extension de la théorie, sans pour autant relever de cette dernière, dans la mesure où les mathématiques ne sont pas consistantes, car les résultats ne cadrent guère avec les principes de base. Un autre exemple d'un usage incorrect de la théorie peut s'observer dans le cadre de la relativité spéciale. Étant donné que la théorie est plus facile à comprendre, cela nous permet de clarifier notre point. On se doit pourtant d'être clair sur ce qui fait partie de la théorie et sur ce qui n'en fait pas partie. La déclaration "un observateur traversant l'horizon de l'événement et perdant tout contact avec le reste de l'univers fait partie de la physique" est un contre-exemple pour la compréhension correcte des théories. Des déclarations incorrectes s'appuyant sur des théories peuvent générer des obstacles à l'avancée de la science. Pour éviter ce piège, il est indispensable de comprendre les fondations philosophiques de la science, et il est important de savoir comment démêler les assomptions cachées d'une théorie.

Key words: Mathematical Assumptions and Scientific Theories; Special and General Relativity; Black Holes; Quantum Mechanics.

I. THE MEANING OF A SCIENTIFIC THEORY

A mathematical system is a collection of arbitrary self-consistent statements. One can have different mathematical systems, with statements that conflict with each other, as long as each system is consistent. The word "theory" is defined as[1] "systematically organized knowledge applicable in a relatively wide variety of circumstances, especially a system of assumptions, accepted principles, and rules of proce-

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dure devised to analyze, predict, or otherwise explain the nature or behavior of a specified set of phenomena." A scientific theory is a mathematical system along with experimental or observational verification.[2] There are two parts to a scientific theory. One is the mathematical system, and the other is the experimental or observational verification. These two parts are not symmetrical. If one demonstrates a single inconsistency with a mathematical system, the entire system is not valid. On the other hand, experimental or observational verification (including measurements) can only be partial, due to Gödel's incompleteness theorems. As long as one has some verification, it is good enough, even if one can find evidence against the theory. The theory must say how one can obtain verification and falsification. If no verification is possible, in principle, then the theory is not valid.

Gödel was a friend of Einstein at Princeton. He proved that within a formal system questions exist that are neither provable nor disprovable based on the axioms that define the system. This is known as Gödel's undecidability theorem. He also showed that in a sufficiently rich formal system in which decidability of all questions is required, there would be contradictory statements. This is known as his incompleteness theorem. In establishing these theorems, Gödel showed that there are problems that cannot be solved by any set of rules or procedures. Gödel used very complicated mathematics to prove that if one has a consistent logical system, such as a theory of numbers, there will always be a true statement that one cannot prove from the axioms.

Applying Gödel's ideas to a physical theory, where predictions can be considered as theorems confirmed by observation or experiment, the conclusion is that there will always be observations that cannot be explained by the theory, and these observations do not invalidate the theory, as would a demonstrated mathematical inconsistency. A false prediction does not invalidate a theory. The false prediction of Newtonian gravitation (NG) of the orbit of Mercury does not invalidate the theory, but merely limits its accuracy. Gödel's work shows that all physical theories have regions where they are not valid.

A physical theory is a logical, self-consistent system that agrees with observation. One can discuss physical concepts only within the framework of a theory, and not confuse concepts from an old theory with the new theory. In addition, one has to be on guard for preconceived ideas or ideas being developed, and address them explicitly.

II. EXAMPLE OF A HIDDEN ASSUMPTION CAUSING CONFUSION

Consider the concept of equilibrium. In classical mechanics, equilibrium at a time t means that the sums of the forces and torques are zero at the time t. When one discusses the theory of special relativity (SR) certain things need to be defined within the theory.

A fundamental concept is Lorentz covariance. Looking at the equilibrium of an object at rest, noting that the sums of forces and torques are zero at a time t, one can view this from another inertial frame, as long as the correct transformations[3] are performed. The result will be that the sums of forces and torques have to be evaluated at different times, where the times are the Lorentz transformations of the times in the proper frames.

The idea that equilibrium must be evaluated at different times was very confusing. Many physicists thought that equilibrium is defined in SR similar to Newtonian mechanics, namely, all the forces and torques at the same time, and, as a result, had to invent many strange ideas. von Laue[4] said that there is an energy current perpendicular to the velocity of the object caused by forces parallel to the velocity, in order to explain how it is possible that an object can be at equilibrium even if the sum of forces and torques is not zero at a time t' in a moving system. It is interesting that von Laue[4] published his idea a few years after the publication of SR, and physicists accepted it uncritically for decades until its refutation.[5] Today one cannot find any reference to von Laue's idea. It may be useful to study this part of the history of science. The error was the adherence to a concept, equilibrium, beyond the valid range by the theory SR. Today some people make a similar mistake, adhering to a concept, covariant transformation between moving systems, beyond the valid range of the current theory, general theory of relativity (GR). This mistake will be discussed below.

The hidden assumption that many physicists held was that the definition of equilibrium means one looks at the sum of the forces and torques at the same time. This statement is not part of the postulates of SR. The system is at equilibrium in the moving system as well, as no work is done. The hidden assumption leads to the introduction of many wrong ideas and confusion. Once one understands this hidden assumption, it can be rejected. The correct statement is instead that equilibrium in a moving system is the covariant transformation of the equilibrium equations from the rest frame, which implies that one does not measure the forces and torques all at the same time in the moving system.

NG is a valid theory, for the mathematical system is consistent, and experiment and observation verify the theory. The orbit of Mercury is evidence against NG, but this does not render the theory invalid. GR has a different mathematical structure, and so is a very different theory. Observations support GR. This does not render NG obsolete, however. GR must have results that agree with NG for those areas that NG has been verified; that is, the weak field nonrelativistic approximation to GR is NG.

Here is another example where, although the logical basis of the theory is sound, but since the observational verification is impossible in principle, the theory does not qualify as a scientific theory. This is the belief in a person's soul going up to Heaven after the person has died. Theologians have worked out the logic over the centuries. However, since no one can observe Heaven, except the dead, people reject the idea of Heaven, saying it is not a scientific concept.

This human failing, to discuss ideas in a physical theory using concepts from an earlier theory, is an issue one needs to be concerned with. Another example is saying an atom is mostly empty space.[6] Here again the teacher is thinking of an atom using Newtonian ideas rather than the correct ideas from quantum mechanics (QM). One needs to stress the fundamental principles.[7]

In summary, a scientific theory requires complete mathematical consistency, and the ability in principle to perform experiments or observations to verify the theory. These conditions may be violated during the development of a theory, as one searches for hypotheses that can explain things. Unfortunately, people sometimes get overenthusiastic, and think that the hypothesis is actually a valid theory. This error can lead to serious philosophical issues.

The mathematical part of a theory consists of a minimal set of initial statements, the postulates. The danger comes when one unconsciously add postulates, thereby creating a theory different from the original theory. Any possible hidden assumptions need to be carefully exposed and discussed. Confusion arises from thinking a hypothesis is a theory, or from a hidden assumption that is not correct.

III. BLACK HOLES

There is confusion about the definition of a black hole (BH). Before discussing a BH, let us look at a well-known BH, the one at the center of our galaxy.[8] The article states "the BH, known as Sagittarius A*, is a certified monster, containing about 4×10^6 times the mass of our Sun." Begelman[9] also discussed evidence for the existence of BHs. We observe BHs by looking at the spectra of objects falling into the BH, and comparing with theory. This complicated and beautiful topic is not part of this paper, except to say that observation seems to match theory.

The idea of an object with gravity strong enough to prevent light from escaping was proposed[10] in 1783. An object needs a minimum velocity to escape from the gravitational attraction of a planet. Once the mass of a star is large enough so that the escape velocity is equal to or greater than the speed of light, in SR no object can escape the star, not even light, as no speed can be greater than the speed of light.

In GR, a BH has an entirely different explanation. The basic equation of GR is $G_{\mu\nu}=8\pi T_{\mu\nu}$, where $G_{\mu\nu}$ is the geometry tensor and $T_{\mu\nu}$ is the energy tensor. Geometry is not Euclidean. Time is slowed down in a gravitational field. A space shuttle will measure slowing of time as it descends from space to Earth by a measurable amount. This is critical for the use of gravitational positioning systems (GPSs).

Imagine the space shuttle landing on an object the size of a small planet, but the mass of a star. The gravitational field will be very large and so that the time dilation will be huge. Distant observers noting the descent will say that the shuttle slowed down.

In an extreme case where the mass of the star is large enough, the time dilation will be so large that the shuttle will never get to the surface. Time has stopped. This is what a BH is in GR. Misner *et al.*[11] gave detailed calculations for objects falling toward large masses, including a BH. A distant observer will see signals from the falling observer redshifted as the objects gets closer, and the signal intensity decreases. This intensity approaches zero in the limit as the object reaches the event horizon, EV. The distant observer never sees the object crossing the EV. The theoretical limit of time to reach the EV is infinite. Misner *et al.*[11] showed that for normal-sized BHs, the actual limit, the time when the redshifted object cannot be seen, is about 1 μs. On the other hand, the falling observer notes that he reached the EV in finite time.

To clarify this, let us use the words of Misner *et al.*[11] "The star, having exhausted its nuclear fuel, contracts inward. For realistic density distributions, the stellar core falls rapidly inward on itself, and the outer envelopes trail behind. In the idealized spherical case, the star's surface falls through its gravitational radius (the horizon, the end of communication with the exterior) in a short lapse of time. From an external vantage point, the star requires infinite time to reach the horizon, becoming black exponentially with e-folding time $\sim10^{-5}M/M_\odot$. From the star's interior vantage point, within a short proper time $\sim10^{-5}M/M_\odot$ after reaching the horizon, a singularity is reached." Misner *et al.*[12] quoted Wheeler saying in 1964 that singularities signal a breakdown of "classical" GR. The word classical is unnecessary, for this means GR without QM; but as of today, there is no GR with QM except as hypotheses.

This is confusing because of a certain lack of symmetry. Normally in both SR and GR, if events in one coordinate system S are observed in another system S', one can say that the same events observed in S' can be observed in S. For the case of the observer falling through the EV, where S is the system of the distant observer and S' the system of the falling observer, the symmetry is broken when the observer in S' crosses the EV, for S cannot see the crossing.

This symmetry breaking should not be of any concern. Consider the case of refraction of light entering water according to Snell's law, $\sin\theta=n\sin\theta'$. For most angles, an observer in the air can view both beams, the beam in the air and in the water; likewise, an observer in the water can see both beams. However, for angles in the air less than or equal to the critical angle, the observer in the air can see both beams, but not the observer in the water. As an eagle approaches the water, it flies at a low angle, putting its talons forward. It can see a fish, but the fish cannot see the eagle. The critical angle breaks the symmetry between the light beams in air and water.

Actually, for the BH case, in a sense the symmetry is not broken. S cannot see S' crossing the EV, and S' cannot see S once it crossed the EV.

One sees how different a BH is in GR and in SR. Most discussions of a BH in GR refer incorrectly to the old idea of a BH as a mass large enough so that the escape velocity is equal to the speed of light, which is the explanation in SR. The concept of time dilation does not exist in SR. One needs GR in order to properly understand what a BH is. One needs to think of the slowing of time, a fundamental relativistic concept, and not only of the gravitational force of attraction.

As soon as the observer enters the BH, the meaning of time and radial distance becomes interchanged, as Misner *et al.*[11] said. An observer will continue falling, just as on Earth one keeps moving forward in time. All timelines crossing the EV reach the center.

There are issues with the solution of GR in the S' system. One is the singularity at the center of the BH, as Penrose and Hawking[13] showed. A singularity is a mathematical inconsistency, such as a division by zero. Since all timelines crossing the EV end at the singularity, all solutions of GR of timelines crossing the EV are not solutions of GR, similar to saying wave solutions not satisfying boundary conditions are not solutions of the wave equation.

The singularity at the center is a genuine singularity, not to be confused with the singularity at the EV, which can be removed by a coordinate transformation.

The other issue is an inconsistency. This inconsistency is only for the BH case, not for objects falling toward massive objects that are not BH. This inconsistency can be clarified as follows. External observers can establish a law of nature that no object can reach the EV. The observer approaching the BH notes that he actually crossed the EV. This contradicts the principle of equivalence, the fundamental postulate of GR, which is that the laws of physics are the same in all moving systems—the outcome of an experiment is independent of the frame of reference. From a frame of reference far from the BH, there is one outcome; and from another frame of reference, moving toward the BH, there is a different outcome. It certainly violates the spirit of equivalence.

One solution is to say that a BH cannot exist, as do Mitra,[14] and Leiter and Robertson.[15] As an object collapses under gravitation, the relativistic time dilation slows down the collapse so that it takes an eternity to reach the final state of a BH, an eternally collapsing object. Crothers[16] looked at the issue from a different point of view, concluding that a BH cannot exist. These ideas are controversial. In this paper, the assumption is that a BH actually exists.

Susskind[17] raised another point. According to Susskind,[17] Hawking claimed that the formal solution from the system S' is valid. All information about the object is lost. Susskind[17] correctly was very disturbed by this. If information is lost, no information is available about the past. Barbour[18] stated that the only reality of the existence of the past is the existence of *present* records. If information is lost, it means that the past is lost, i.e., never happened, yet observers did record the falling of the objects, a paradoxical situation. Susskind's solution is his holographic principle, derived from QM, which states that all the information about the "inside" of the BH resides outside. However, a theory of quantum gravitation does not yet exist. Susskind's points demonstrate the difficulties of discussing the solution of GR from the point of view of an observer entering the BH.

The approach in this paper dealing with the conflict between these two solutions is simply to reject the solution of GR from the falling observer's viewpoint. This avoids the mathematical and philosophical inconsistencies. When one rejects a solution, it does not mean he rejects the entire theory, similar to rejecting solutions for a vibrating string not satisfying boundary conditions.

If one accepts this idea, rejecting the solution of the falling observer entering the BH in finite time, one sees that a BH cannot be defined in GR by saying nothing can leave, for GR does not discuss anything entering. The only way a BH can be defined is saying objects take an infinite time to reach the BH. A BH can be defined by using ideas only from GR, and not from earlier physics. This is challenging for many people, as the classical idea of a BH is centuries old. People think they understand relativity when they are really thinking the old way, and not using the concepts of the current theory.

The statement that at crossing the EV the meaning of time and radial distance becomes interchanged does not make sense. One normally views time as something that flows on, and cannot imagine how time could become distance. Well, if this is what the equations are, this strange interpretation must be accepted. However, these equations are not valid everywhere inside the BH, as they are not valid at the center, and so one does not have to accept this nonsensical sounding explanation. Furthermore, Misner *et al.*[11] told us that the observer crossing the EV cannot communicate to the outside world and then very quickly will be killed. The question is how far one has to push our imagination to entering the BH. Why not stop at the EV, and ignore possible interpretations of the inside of the BH?

Misner's book was written a long time ago. The mathematics is clear, easy to read, and rigorous. The physical interpretations of the observer inside the BH are merely words that do not make sense, and are based on hidden false assumptions. The awe people have given to these authors in their extensive mathematics must not overshadow our basic understandings of the meanings of physics. The fact that no one during the past decades has challenged them for their incorrect physical interpretations does not mean they were right. It is not meaningful science to say time can change into space. Scientists understand that time slows down due to gravity, and this is critical in the design of GPSs. Time slowing down is not time becoming space. That interpretation is nonsense.

These discussions invariably say that a quantum theory of gravity will resolve the difficulties. This statement is false. Scientists cannot say what a future theory will do, but only that a future theory *may* resolve the difficulties. Furthermore, since one cannot perform observations of the inside of the BH, why the discussion from the point of physics? It is fine to do the mathematics. It is not fine to say this is physics.

IV. MORE EXAMPLES OF HIDDEN ASSUMPTIONS

The hidden assumption physicists have is that *by working hard, people will eventually get to understand the structure of the universe, and to be able to resolve the difficulties in current theories.* This assumption is beautiful and inspiring. It helps generate support for research, and encourages new generations of students to enter the field of physics to try to understand and resolve the mysteries. What is strange is that everyone knows that this assumption is wrong. Gödel's incompleteness theorems clearly state that one cannot know everything. This means that one has to be careful to distinguish between valid theories and working hypotheses. For example, there is a conflict between GR and QM. It is possible that we will *never* resolve this conflict. When one discusses certain issues in physics, one tends to say that a future theory of quantum gravitation will explain this issue. This is not the correct thing to say discussing a theory of physics. The introduction of terms from QM when discussing GR only confuses the issue.

A variant of this assumption is the assumption that *one can understand the universe beyond current valid theories*. Talk about what the universe is like based on some form of quantum gravitation is very prevalent, but, unfortunately, confuses everything, by mixing discussion of valid theories with hypotheses. In a similar vein are the discussions of religion and science, where religion explains the universe beyond current valid theories.

Another hidden assumption is that *one can imagine going anywhere*. One can imagine going into the center of the Sun, even though there is no way to design a probe that can descend into the solar interior and send out information. One can imagine going to distant galaxies millions of light years away, even though one cannot design a system to travel for millions of years. One imagines being small enough to look at moving molecules. Maxwell's Demon, from 19th century thermodynamics, was an imaginary intelligent being who sat at a hole in a balloon as the air rushed out, was able to close the door as molecules went out, and opened the door to let molecules in.

Let us be careful not to be carried away with these mental images. Imagine Maxwell's Demon inside a vial of liquid helium, watching the atoms zip by. As the helium is cooled below the lambda temperature, the helium becomes a superfluid, and the meaning of position is gone. Above the lambda temperature one can speak about the individual positions of the atoms; below this temperature, one can only say that the atoms are in the vial. This shows that one cannot imagine being as small as an atom, and observing the atoms.

V. CONCLUSIONS

One must not try to say that GR is valid in regions where it is known not to be valid. Yes, one can use our imagination and try to work out possible extensions of the theory as part of efforts to develop a more inclusive theory. This is what Einstein did in working with NG and other theories that lead him to GR. Do not say to the public that these possible extensions are part of the theory and a description of nature.

Do not be lulled into a false sense of confidence that there is no harm done discussing physics inside a BH, for, after all, the world's physicists have all agreed for decades that physics inside a BH is meaningful physics. The point is that since physics based on current accepted theories and philosophy of physics cannot describe the inside of a BH, our insistence that one can indeed describe this region has the potential to do great harm to our scientific enterprise as a whole.

GR is a valid scientific theory. The mathematical basis is sound. Agreement with observations and experiments is excellent. A BH is an object predicted by GR as an object with mass large enough to stop time for anything approaching it. Scientists are currently performing observations trying to understand more about the phenomena of objects falling toward a BH and the energy released as the objects approach the BH. Once we start thinking thought experiments of objects actually crossing the EV, we have left the area of established theory and entered the region of speculation toward future theories. These speculations can be fruitful as long as we do not confuse them with GR. There are problems with GR, especially dealing with quantum phenomena. It is useful to try to work toward development of extensions to GR, just do not confuse this with GR itself.

Proper understanding of the nature of scientific theories and their development is critical to the advance of science. For many decades, theoretical physics has been treading water. The focus on fundamentals, along with the ability to communicate the ideas, will hopefully allow science to progress to new and deeper understandings.

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