

Here are definitions of the math symbols \sum for summation of terms and \prod for multiplication of factors. These are capital Greek letters.

$\sum_{i=1}^5 (2+i)$ means sum terms, starting with $i = 1$ and finishing with $i = 5$.

We do not write $i = 5$ on the top of the sigma, as we know we sum over i .

$$\sum_{i=1}^5 (2+i) = (2+1) + (2+2) + (2+3) + (2+4) + (2+5)$$

Same with the product sign. We multiply factors.

$$\prod_{i=1}^5 (2+i) = (2+1) \cdot (2+2) \cdot (2+3) \cdot (2+4) \cdot (2+5)$$

Definition of $n!$:

For $n > 0$:

$$n! = n(n-1)(n-2) \cdots 1$$

$$= \prod_{i=0}^{n-1} (n-i)$$

$$0! = 1$$

This is the definition of $n!$ for $n = 0$.

BY THE PERMUTATIONS of the letters abc we mean all of their possible arrangements, where order matters ($ab \neq ba$)

abc
 acb
 bac
 bca
 cab
 cba

There are 6 permutations of three different things. As the number of things (letters) increases, their permutations grow astronomically. For example, if twelve different things are permuted, then the number of their permutations is 479,001,600.

Now, this enormous number was *not found by counting them*. It is derived theoretically from the **Fundamental Principle of Counting**:

If something can be chosen, or can happen, or be done, in m different ways, and, after that has happened, something else can be chosen in n different ways, then the number of ways of choosing both of them is $m \cdot n$.

For example, imagine putting the letters a, b, c, d into a hat, and then drawing two of them in succession. We can draw the first in 4 different ways: either a or b or c or d . After that has happened, there are 3 ways to choose the second. That is, to each of those 4 ways there correspond 3. Therefore, there are $4 \cdot 3$ or 12 possible ways to choose two letters from four.

ab	ba	ca	da
ac	bc	cb	db
ad	bd	cd	dc

ab means that a was chosen first and b second.

ba means that b was chosen first and a second.

The number of permutations of n different things taken n at a time is $n!$.

In **permutations**, the order is all important -- we count abc as different from bca . But in combinations we are concerned only that a, b , and c have been selected. abc and bca are the same combination.

Here are all the **combinations** of $abcd$ taken three at a time:

abc abd acd bcd .

Since the **order does not matter in combinations**, there are clearly fewer combinations than permutations. The combinations are contained among the permutations -- they are a "subset" of the permutations.

What is the sum of all the combinations of n things?

$${}_n C_0 + {}_n C_1 + {}_n C_2 + \dots + {}_n C_n = 2^n$$

The combination is the number of subsets, and the above is the total number of subsets.

Let us look at examples of **permutations**. We have n objects and take r at a time, where order matters.

If $r=1$, then there are n ways to do it, i.e., n ways to take one object out. If $n=5$, then there are 5 ways to do it.

If $r=n$, then there are $n!$ ways to take out n objects. n ways to take the first, $n-1$ ways to take the second, etc. E.g., if $n=r=5$, there are $5!$ ways to take 5 objects from 5 objects. There are 5 ways to take the first, 4 ways to take the second, 3 ways to take the third, and 2 ways to take the fourth.

Suppose we wanted to take 2 objects from 5. There are 5 ways to take the first, 4 ways to take the second object, and 3 ways to take the third. We stop at 3 as we have 2 objects left over. That is, there are $5 \cdot 4 \cdot 3$ ways.

In general there are $n(n-1)(n-2)\dots(n-r+1)$ ways to pick out r objects out of n . We stop at $n-r+1$ as then there are r objects left. We can write

$$\begin{aligned} n(n-1)(n-2)\dots(n-r+1) &= \prod_{i=0}^{r-1} (n-i) \\ &= \frac{n!}{(n-r)!} \end{aligned}$$

Check this out by putting $r=1$ and $r=n$.

In summary, the number of ways of choosing r objects from n , called **permutations**, is

$${}_n P_r = \frac{n!}{(n-r)!}$$

For **combinations**, we do not care about the order of the r objects, and so we divide by the number of ways of arranging r objects, which is $r!$. We have

$$\begin{aligned} {}_n C_r &= \frac{{}_n P_r}{r!} \\ &= \frac{n!}{(n-r)!r!} \end{aligned}$$