

BY THE PERMUTATIONS of the letters  $abc$  we mean all of their possible arrangements:

$abc$

$acb$

$bac$

$bca$

$cab$

$cba$

There are 6 permutations of three different things. As the number of things (letters) increases, their permutations grow astronomically. For example, if twelve different things are permuted, then the number of their permutations is 479,001,600.

Now, this enormous number was not found by counting them. It is derived theoretically from the **Fundamental Principle of Counting**:

If something can be chosen, or can happen, or be done, in  $m$  different ways, and, after that has happened, something else can be chosen in  $n$  different ways, then the number of ways of choosing both of them is  $m \cdot n$ .

For example, imagine putting the letters  $a, b, c, d$  into a hat, and then drawing two of them in succession. We can draw the first in 4 different ways: either  $a$  or  $b$  or  $c$  or  $d$ . After that has happened, there are 3 ways to choose the second. That is, to each of those 4 ways there correspond 3. Therefore, there are  $4 \cdot 3$  or 12 possible ways to choose two letters from four.

$ab$              $ba$              $ca$              $da$

$ac$              $bc$              $cb$              $db$

$ad$              $bd$              $cd$              $dc$

$ab$  means that  $a$  was chosen first and  $b$  second.

$ba$  means that  $b$  was chosen first and  $a$  second.

The number of permutations of  $n$  different things taken  $n$  at a time is  $n!$ .

In permutations, the order is all important -- we count  $abc$  as different from  $bca$ . But in combinations we are concerned only that  $a$ ,  $b$ , and  $c$  have been selected.  $abc$  and  $bca$  are the same combination.

Here are all the combinations of  $abcd$  taken three at a time:

$abc$   $abd$   $acd$   $bcd$ .

Since the order does not matter in combinations, there are clearly fewer combinations than permutations. The combinations are contained among the permutations -- they are a "subset" of the permutations.

What is the sum of all the combinations of  $n$  things?

$${}_n C_0 + {}_n C_1 + {}_n C_2 + \dots + {}_n C_n = 2^n$$

The combination is the number of subsets, and the above is the total number of subsets.

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